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Oktay Surucu

Lying for the Greater Good: Bounded Rationality in a Team

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Lying for the Greater Good: Bounded Rationality in a Team

Oktay Surucu
[oktay.surucu@unive.it]
Advanced School of Economics,
University of Venice

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Abstract

The article is concerned with the interaction between fully and boundedly rational agents in situations where their interests are perfectly aligned. The cognitive limitations of the boundedly rational agent do not allow him to fully understand the market conditions and lead him to take non-optimal decisions in some situations. Using categorization to model bounded rationality, we show that the fully rational agent can manipulate information to help decreasing the expected loss caused by the boundedly rational agent. Assuming different types for the boundedly rational agent, who differ only in the categories used, we show that the fully rational agent may learn the type of the boundedly rational agent along their interaction. Using this additional information, the outcome can be improved and the amount of manipulated information can be decreased. Furthermore, as the length of the interaction gets longer the probability that the fully rational agent learns the type of the boundedly rational agent increases.

Keywords: bounded rationality, categorization, learning.

JEL Classification Numbers: C0, C70, D83.

1 Introduction

In economic literature, one of the most commonly used assumptions about decision makers is full rationality. When faced with an economic decision problem, a fully rational decision maker has the ability to see and understand what is feasible and what is preferable. Furthermore, he is also able to calculate the optimal course of action given these two constraints. This widely used assumption that simplifies economic models has received many criticisms for overlooking real life situations by ignoring cognitive limitations. Wide literature initiated by Amos Tversky, Daniel Kahneman, and their collaborators provides us with experimental evidence that human beings depart systematically from full rationality due to cognitive limitations. These limitations affect their ability to recognize the available information on markets and their ability to compute. Herbert Simon, the originator of the phrase, defines bounded rationality as "rational choice that takes into account the cognitive limitations of the decision-maker-limitations of both knowledge and computational capacity" (Simon 1987).

Boundedly rational agents try to simplify and structure the economic decision process. One of the possible ways to do this is to use categories. The usage of categories is also supported by psychological evidence that people in environments with abundance of information show the tendency to group events, objects or numbers into categories depending on their perceived similarities (Rosch and Mervis 1975). The social psychologist Gordon Allport states that "the human mind must think with the aid of categories. We cannot possibly avoid this process. Orderly living depends upon it"(Allport 1954, pg 20). Both in economic and social psychological literature, there are many studies aiming to explain human behavior using categorization (e.g. see Macrae and Bodenhausen 2000 or Fryer and Jackson 2008).

The following example illustrates one possible way how the categorization process works. Consider a consumer who wants to buy a new television. There are an overwhelming number of available alternatives on the market. In order to make a decision, the consumer has to compare a long list of attributes among all products. These attributes include a wide variety of technical features (e.g. screen size, aspect ratio, resolution, contrast ratio, sound system, dimension, weight, etc.), price arrangements (price of the product, payment schedule, service fees), brand, warranty, product support, delivery service, etc. Unless the consumer is an expert on televisions, he may have difficulties in making decision because of this long list of items to consider for each product on the market. What happens most of the time is that after eliminating the obviously undesirable products (e.g. too expensive products), the consumer categorizes all the remaining products on the market so that in each category there are products with some similar attributes. One possible categorization process works as follows. At each step of the process, the consumer chooses an attribute, attaches some criteria to the attribute and partitions the set of products based on the criteria. Say, for example, he considers the screen size attribute and the criteria he attaches is if it is less than 45 inches or between 45 and 55 inches or larger than 55 inches. In this way, he partitions the products into three sets as "products with screen sizes less than 45 inches", "products with screen sizes between 45 and 55 inches" and "products with screen sizes higher than 55 inches". He continues the categorization process by choosing another attribute-criterion tuple, say resolution and a threshold for resolution. He further refines each set in his partition based on this new attribute-criterion tuple and obtains a new partition. In particular, he divides each of the three sets into two as high-resolution and low-resolution, and ends up with 6 sets (categories) in his new partition (low resolution-small size, high resolution-small size, low resolution-medium size, high resolution-medium size, low resolution-big size, high resolution-big size). Repeating this process for a number of steps, he ends up with a final partition of products.[1] Each category in this partition includes a subset of products on the market having similar features. He chooses one product from each category as a representative and compares all the representatives. Then he considers only the category whose representative gives the maximum utility. The final decision is made among the products in that category. This process may lead to a non-optimal decision since the consumer considers only a small subset of products (the category whose representative gives him the highest utility) rather than the whole set. Furthermore, another feature of categorization is that even if their preferences are perfectly aligned, the decisions made by different individuals may not be the same. The reason is that the final partition for a consumer is most likely to be different than the final partition of another consumer, since it depends on the number of steps and the criteria the individuals use.

The main purpose of the paper is to analyze the interaction between fully and boundedly rational people. More specifically, we focus on situations in which both agents work together in a team and the boundedly rational agent has to make a decision after receiving a message from the fully rational agent. In such setups, although being fully rational, an agent might suffer from possible non-optimal decisions made by boundedly rational agent. We investigate how a fully rational agent can decrease the expected loss due to bounded rationality. We show that this is possible by manipulating information sent to the boundedly rational agent. Furthermore, we focus on what the fully rational agent can infer about the categories used by boundedly rational agent among their interaction and we show that it is possible to decrease the amount of manipulated information.

The following setting about a fully rational boss and his boundedly rational namesake can be considered as a motivating example for our model. The boss, who can be regarded as the principal, is willing to buy arms for hunting animals. Having a criminal record, he does not meet the conditions for registration of arms with the police forces. Therefore he asks his namesake, who does not have any records of criminal commitment, to buy a weapon for him. The namesake, who can be regarded as the agent, has also some connections in the weaponry black market. Therefore he can buy the weapon from either the legal or illegal market. At this point, it is important to note that the problem we are dealing with is not a principal-agent problem, but an instance of team theory initiated by Roy Radner. In principal-agent problems there is a conflict of interest giving rise to agency cost. In our setting, however, this is not the case since the preferences of the boss and his namesake are perfectly aligned.

Our paper takes as a departure point Dow (1991), where an economic decision problem for a boundedly rational agent visiting two stores and searching for the lowest price is modeled. The bounded rationality of the agent comes from his limitations in memory. More specifically, when the agent is in the second store, he cannot remember the exact price in the first store,

[1]The number of steps depends on the degree of the individual's bounded rationality. In the limit case (when the individual is fully rational, say, an expert on televisions), the number of steps is sufficiently large that each category contains only one product (finest partition).

but only remembers to which category it belongs. The agent makes a decision by comparing the price in the second store with the representative of the category to which the price in the first store belongs. Dow (1991) characterizes the optimal categorization. We depart from Dow's setting by introducing a fully rational agent and examining the interaction between the two agents.

Considering a similar setting to Dow's (1991), Chen, Iyer and Pazgal (2005) and Luppi (2006) examine the price competitions in the market and show that fully rational firms can take advantage of boundedly rational consumers. Chen, Iyer and Pazgal (2005) depart from Dow's setting by introducing two different types of consumers: totally uninformed consumers, who only consider buying from a specific store as long as the price is below their reservation value, and informed consumers with perfect memory, i.e., fully rational consumers. They characterize the Nash equilibrium of the game in which firms choose pricing strategies and consumers with limited memory choose their categories. It is shown that having bounded rational agents in the market softens price competition. A similar setting is used by Luppi (2006), where there are rational firms on one side and boundedly rational consumers on the other side of the market. Consumers categorize the price space and make their decision based on their categories. She demonstrates that in the presence of boundedly rational consumers two firms competing a la Bertrand depart from the standard equilibrium and make positive profits. The difference between these two papers and ours comes basically from the difference in the settings. In our case fully rational and boundedly rational agents are working as a team and their common aim is to improve the outcome. In other words, the fully rational agent is not trying to take advantage of the boundedly rational agent like in Chen, Iyer and Pazgal (2005) and Luppi (2006), but he is trying to learn how to deal with the latter one in order to achieve the common goal.

Another literature strand to which this paper refers is the field of Information Transmission. Crawford and Sobel (1982) analyze costless strategic communication between a better-informed, fully rational sender and a fully rational receiver. The sender categorizes the support of messages and sends the category to which the realized message belongs instead of sending the real value. This situation arises because the players' preferences are not perfectly aligned. The receiver, after reading the signal, takes an action that affects both his and the sender's payoffs. They show that as the preferences become more aligned, the number of categories the sender uses increases, i.e., the signal becomes more informative. The main difference from our model relies on differences in assumptions: full rationality of both agents and differences in preferences.

Although there have been many studies in economic literature on bounded rationality, studies on interaction between fully and boundedly rational agents are limited in number. To our knowledge all these studies concern with how fully rational agents take advantage of boundedly rational agents (see Rubinstein 1993, Piccione and Rubinstein 2003, Eliaz and Spiegler 2006). The main novelty of our paper lies in our team approach. Both type of agents work together to decrease the inefficiency caused by bounded rationality since their preferences are perfectly aligned.

Another interpretation of our model could be done by using the concept of interpreted signals rather than bounded rationality. This concept, introduced by Hong and Page (2009), is based on the assumption that people filter reality into a set of categories. Hong and Page call the predictions that agents make about the value of the variable of interest by using their own categories as interpreted signals. They state that "... two agents' signals differ if the agents rely on different predictive models. This can only occur if agents differ in how they categorize or classify objects, events or data, if agents possess different data, or if agents make different inferences." In our model, we can think that the interpreted signal of the boss and his namesake may differ due to their different ways to categorize the real world. In this case, the action taken by the namesake may cause a loss for the boss because the good bought by his namesake might be less valuable for the boss than the alternative. In order to decrease this expected loss, the boss manipulates the information he sends to his namesake. Moreover, it might be possible to decrease the amount of manipulated information, since the boss might infer the categorization of his namesake among their interaction.

The organization of the paper is as follows. Section 2 describes our two-period toy model, gives the details of learning mechanism and presents results obtained using myopic approach. Section 3 recaptures the results using a farsighted approach and Section 4 concludes.

2 A Toy Model

We consider a two-period decision problem, in which a fully rational boss wants to buy a product in each period. There are two markets having a huge number of alternatives for the product. The first market is more complex than the second one. A possible explanation for this could be that the first market is a legal market with many regulations and the second market is an illegal one with less complexity. The boss can only observe the products in the first market but cannot perform any transaction since he does not have access to neither of the markets. Therefore he asks his boundedly rational namesake, who has access to both markets, to compare products in the two markets and buy from one. However, cognitive limitations of the namesake do not allow him to fully understand the complex (first) market. Being aware of his limitations, he categorizes the price space for the first market to simplify the decision process and uses the representatives of his categories in order to compare the prices in two markets. The objective of the boss is to minimize the expected loss due to the cognitive limitations of his namesake.

It is common knowledge that the boss is fully and the namesake is boundedly rational. It is also known by both parties that the bounded rationality of the namesake is due to his limited ability in understanding the first market. It should be noted that for simplicity we consider only a single number (price) for a product, but in fact this is a combination of many elements, like the type, quality, brand, and age of the product, length of the warranty, payment arrangements and service fees. It is the multiplicity of such items that makes the namesake unable to fully understand the first market. However, the number of elements that are embedded in prices of the second market is less than those of the first market. In case of an illegal market, for example, there are no warranties, no payment arrangements, no service fees, etc. This is what makes the first market more complicated than the second market. In other words, this is the reason why the namesake is unable to fully understand the first market whereas he understands the second market. Being aware of his limitation, the namesake fully trusts his boss. This is because he knows that their preferences are perfectly aligned and that the boss is fully rational, i.e., that the boss does not have any limitations in understanding the market. Furthermore, the namesake is aware of the fact that the boss may lie to him. However he knows that the reason for that is not that the boss wants to take advantage of him but to improve the outcome. Finally, the boss knows that his namesake fully trusts him.

In the first period, the boss observes the price on the first market, p_1^1, and then reports a price to his namesake, p^1 (not necessarily the true observed value). Receiving the report, the namesake understands to which category the reported price belongs. Then he compares the representative of that category with the price on the second market, p_2^1, and decides from which market to buy. Note that he may take a non-optimal action since he uses the representative instead of the realized price for the product in the first market. Finally, he informs his boss about the price on the second market. Therefore, the boss is able to understand whether the decision was optimal or not.

At the beginning of the second period, the boss updates his beliefs about the namesake's categories by looking at the realized prices on both markets and the action of the namesake. Then the first period is repeated. The notations used for the second period are as follows: p_1^2 stands for the realized price on the first market, whereas p_2^2 is the price on the second market, and p^2 is the reported price.

We assume that prices on both markets are independent and distributed uniformly between 0 and 1. There are three possible types for the namesake. All types use two categories, namely, they all partition the price space in two. In order to do that they choose a cutoff price level. Prices lower than the cutoff level belong to the first category (low) and prices higher than the cutoff belong to the second category (high). The representative of each category which is used to compare with the price on the second market is the median of that category. Types differ in their choices of cutoff price level. Type-1 uses $1/4$ as the cutoff level and the representative price of his low category is $1/8$, whereas it is $5/8$ for his high category. Type-2 uses $1/2$ as the cutoff level, thus $1/4$ and $3/4$ are the representatives for his low and high categories, respectively. Finally, type-3 who uses $3/4$ as the cutoff level, has $3/8$ and $7/8$ as the representatives for his low and high categories, respectively. The prior belief of the boss is that all types are equally likely.

Given the number of categories and price distribution, type-2 uses optimal categories. The cutoff that is used by type-1 is lower than what it should be. It can be thought that type-1 believes that the mean of prices is low. On the contrary, type-3 uses a cutoff higher than the optimal. With the same logic, he can be thought as a person who believes that the mean is high. The distances of cutoffs of type-1 and type-3 from the optimal level of cutoff $(1/2)$ are the same but in reverse directions. This is to say that type-1 and type-3 behave symmetrically.

The objective of the boss is to minimize his expected loss caused by bounded rationality. His action is the price that he reports to the namesake. There are 4 different types of available action that are given in Table 1. For example, if the boss chooses to report a price in $[0, 1/4]$, then all the types consider their low categories, and use $1/8$, $1/4$, $3/8$ as representative, respectively.

	type-1	type-2	type-3	used prices
$p^1 \in [0, \frac{1}{4}]$	L	L	L	$\{ \frac{1}{8}, \frac{1}{4}, \frac{3}{8} \}$
$p^1 \in [\frac{1}{4}, \frac{1}{2}]$	H	L	L	$\{ \frac{5}{8}, \frac{1}{4}, \frac{3}{8} \}$
$p^1 \in [\frac{1}{2}, \frac{3}{4}]$	H	H	L	$\{ \frac{5}{8}, \frac{3}{4}, \frac{3}{8} \}$
$p^1 \in [\frac{3}{4}, 1]$	H	H	H	$\{ \frac{5}{8}, \frac{3}{4}, \frac{7}{8} \}$

Table 1: Action Space

In this section we consider a myopic approach. That is, we assume that the boss is only concerned with the expected loss of the current period, not with the aggregate expected loss. A farsighted approach is considered in the following section. Table 2 shows the expected loss for each possible combination of price realizations on the first market (p_1^1) and actions taken by the boss. Each number in bold gives the minimum expected loss for the relevant price realization. Given the myopic approach, the action that corresponds to each bold number is the optimal choice of action for the boss for the relevant price realization. For example, if the boss observes a price on the first market that belongs to interval $[0, 1/8]$, he will report a price that belongs to interval $[0, 1/4]$. At this point we make another assumption about the boss. We assume that he prefers to tell the truth whenever it is among the optimal actions. This assumption together with the fact that $[0, 1/8] \subset [0, 1/4]$ (truth-telling is among optimal actions) imply that the boss reports the true value in this case. However, if $p_1^1 \in [1/4, 3/8]$ it is optimal to report $p^1 \in [0, 1/4]$. In this case, the reported price is less than its true value (the boss under-states the price). The other case in which the boss lies is when $p_1^1 \in [5/8, 3/4]$. The reported price in this case is $p^1 \in [3/4, 1]$, i.e., it is higher than its true value (the boss over-states the price).

Observed Price\ Report	$p^1 \in [0, \frac{1}{4}]$	$p^1 \in [\frac{1}{4}, \frac{1}{2}]$	$p^1 \in [\frac{1}{2}, \frac{3}{4}]$	$p^1 \in [\frac{3}{4}, 1]$
$p_1^1 \in [0, \frac{1}{8}]$	**9**	29	57	93
$p_1^1 \in [\frac{1}{8}, \frac{1}{4}]$	**3**	15	35	63
$p_1^1 \in [\frac{1}{4}, \frac{3}{8}]$	**3**	7	19	39
$p_1^1 \in [\frac{3}{8}, \frac{1}{2}]$	9	**5**	9	21
$p_1^1 \in [\frac{1}{2}, \frac{5}{8}]$	21	9	**5**	9
$p_1^1 \in [\frac{5}{8}, \frac{3}{4}]$	39	19	7	**3**
$p_1^1 \in [\frac{3}{4}, \frac{7}{8}]$	63	35	15	**3**
$p_1^1 \in [\frac{7}{8}, 1]$	93	57	29	**9**

Table 2: Expected Loss (common multiplier: $\frac{1}{6\times 8^3}$)

Table 2 results in the following reaction function:

$$R(p_1^1) = \begin{cases} \text{report } p^1 \in [0, \frac{1}{4}] & \text{if } p_1^1 \in [\frac{1}{4}, \frac{3}{8}], \\ \text{report } p^1 \in [\frac{3}{4}, 1] & \text{if } p_1^1 \in [\frac{5}{8}, \frac{3}{4}], \\ \text{report the true price} & \text{otherwise.} \end{cases} \tag{1}$$

Under-statement of the price occurs only if $p_1^1 \in [1/4, 3/8]$ and receiving this report all types use their low (L) categories (see Table 1). However, if $p_1^1 \in [1/4, 3/8]$ and the boss reports the true value of the price rather than under-stating, type-1 uses his high (H) category whereas type-2 and 3 stick to their low (L) categories. So, it is only type-1 who is affected by under-statement. Since the boss prefers to tell the truth whenever it is among the optimal actions and under-statement does not affect other types, the boss uses this strategy only if type-1 is among possible types when the observed price belongs to interval $[1/4, 3/8]$.

Over-statement of the price occurs only if $p_1^1 \in [5/8, 3/4]$. By the same reasoning above, over-statement affects only type-3, not others. Therefore, the boss uses this strategy only if type-3 is among possible types when $p_1^1 \in [5/8, 3/4]$. Otherwise, he prefers to report the truth.

Figure (1) represents the reaction function of the boss. Here, we can observe that the behavior of the boss is symmetric around 1/2. The arrow on the left represents under-statement and in case of under-statement only type-1 switches category, whereas the arrow on the right represents over-statement and only type-3 switches category in this case. As noted earlier, these types behave symmetrically which results in symmetric behavior of the boss.

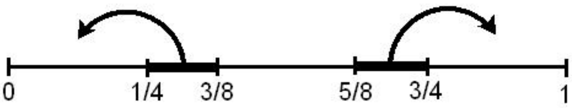

Figure 1: Reaction Function

At the end of the first period, the boss updates his beliefs by looking at the prices realized in both markets and the action taken by the namesake. To see how this works let us consider the following example. Say, $p_1^1 \in [0, 1/8]$, $p_2^1 \in [1/8, 1/4]$. Given the price on the first market, the boss reports the true value (see (1)). In this case, the representative price is 1/8 for type-1, 1/4 for type-2 and 3/8 for type-3. The namesake, comparing the representative price with the price on the second market, buys the good from the first market if he is of type-1 and buys from the second market if he is of type-2 or type-3. If the product is bought from the first market, the boss understands that his namesake is of type-1 and updates his belief such that with probability 1 the namesake is of type-1. If instead, it is bought from the second market, the boss updates his belief such that with probability 1/2 the namesake is of type-2 and with probability 1/2 the namesake is of type-3.

Figure 2 summarizes the learning process at the end of period-1. Numbers in bold stand for the numbers of possible types of the namesake. The boss starts with three possible and equally likely types. The probability that he learns the exact type, i.e., that the number for possible types is 1, at the end of the first period is $\frac{3}{32} = 0.09375$. The probability that the number of possible types decreases to 2 (elimination of one type) is $\frac{3}{16} = 0.1875$, and finally the probability that the boss learns nothing is $\frac{23}{32} = 0.71875$.

The boss starts the second period with updated beliefs. The objective is again to minimize the expected loss caused by bounded rationality. When type-1 is among possible types and

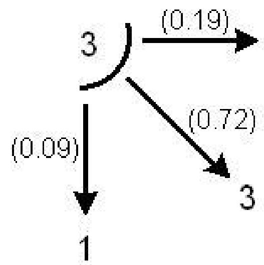

Figure 2: Learning Process, 1st Period

the observed price on the first market in the second period (p_1^2) belongs to the interval $[1/4, 3/8]$, he uses the under-statement strategy described above. Furthermore, when type-3 is among possible types and $p_1^2 \in [5/8, 3/4]$, he uses the over-statement strategy. In all the other cases he reports the true observed value. The reaction function for the second period coincides with the one for the first period (1) if both type-1 and type-3 are among possible types.

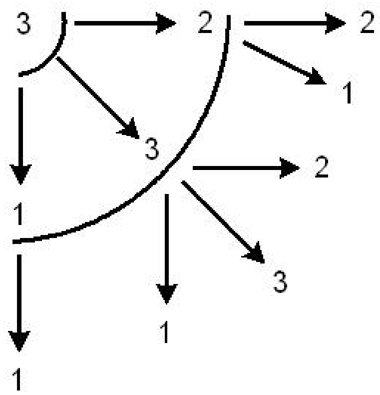

Figure 3: Learning Process, 2nd Period

Figure 3 summarizes the learning process for the whole game. If the boss figures out the exact type of the namesake (arrives to node 1) at the end of the first period, there is nothing left to learn and he continues the second period with the relevant strategy. If he arrives to node 2 at the end of the first period, the learning process continues and he might either figure out the type and arrive to node 1 or not learn anything and stay in node 2. If he does not learn anything about the type at the end of the first period (stays at node 3), there are three possibilities for the second period. He might figure out the exact type and arrive to node 1, or he might eliminate only one possible type and arrive to node 2, or he might not learn anything and stay at node 3. The overall probability that the boss figures out the exact type of the namesake by the end of the game is 0.19238, that he eliminates only one possible type is 0.29102 and that he does not learn anything is 0.51660.

The transition matrix of the learning process is given in Table 3. It is a finite Markov Chain and has three ergodic states. According to the Theorem by Kemeny and Snell (1976), the probability after n steps that the process is in an ergodic state tends to 1, as n tend to infinity. This means that if the game is repeated for n periods the probability that the boss learns the exact type of the namesake tends to 1 as n gets larger.

possible types	{1,2,3}	{1,2}	{1,3}	{2,3}	{1}	{2}	{3}
{1,2,3}	0.71875	0.08333	0.02083	0.08333	0.04167	0.01042	0.04167
{1,2}	0	0.84375	0	0	0.07813	0.07813	0
{1,3}	0	0	0.75000	0	0.12500	0	0.12500
{2,3}	0	0	0	0.84375	0	0.07813	0.07813
{1}	0	0	0	0	1	0	0
{2}	0	0	0	0	0	1	0
{3}	0	0	0	0	0	0	1

Table 3: Transition Matrix

The relationship between the number of periods and the probability of learning the exact type is given in Table 4. The probability increases in the number of periods, and it becomes almost 1 after 30 periods.

n	p
5	0.46344
7	0.60419
10	0.75543
15	0.89388
20	0.95465
30	0.99178

Table 4: Number of periods/probability

A crucial point to be noted is that in this section we use a myopic approach to solve the optimization problem. This means that we assume the boss is concerned only with the expected loss of the period he is in. Whereas with a farsighted approach, he considers the overall expected loss that is the sum of discounted expected losses. However, both approaches yield the same results with the given available types. In this setting, a manipulated message affects only one type, while other types stick to their category that they would consider without the manipulated message. In other words, a strategy that needs to be used in order to decrease the expected loss caused by one type does not conflict with the strategies that need to be used for other types. For example, the under-statement strategy is used whenever type-1 is among possible types. The fact that type-2 and/or type-3 are among possible types does not change this strategy, because it induces only type-1 to change his category, not the other types.

Therefore, the boss can continue to use the reaction function given in (1) even if he knows the exact type of the namesake. It should be noted that if he does so, he might report a manipulated price although reporting the true value is also among optimal actions. Even tough this violates our assumption that the boss prefers reporting the truth whenever it is possible, it yields the same expected loss for the boss. This fact ensures that he can use the same reaction function for each period no matter if he is farsighted or myopic. In the following section we show that myopic and farsighted optimizations do not always coincide.

3 Farsighted Approach

In this section, we consider a farsighted approach. That is, we assume that the objective of the boss is to minimize the sum of discounted expected losses. We modify the model by changing the possible types. Here, we assume that the namesake has two possible types. The first type uses two categories (low and high) and his cutoff price level is 1/3. Therefore he uses 1/6 as the representative for low category (L) and 2/3 for high category (H). The second type uses three categories (low, medium and high) and his cutoff price levels are 1/3 and 2/3. Thus 1/6, 1/2 and 5/6 are the representative prices for his low (L), medium (M), and high (H) categories, respectively. The prior belief of the boss is that both types are equally likely.

In this setting, the boss can choose his strategy among three different types of action, that are represented in Table 5. If he reports a price belonging to $[0, 1/3]$, both types use low categories and 1/6 as representative price. If he reports $p^i \in [1/3, 2/3]$, then type-1 uses his high category and 2/3 as his representative for the first market price, and type-2 uses his medium category and 1/2 as the representative ($i \in \{1, 2\}$ represents the period). Finally, if the boss reports $p^i \in [2/3, 1]$, both types will use high categories and type-1 uses 2/3 whereas type-2 uses 5/6 as representative price.

	type-1	type-2	used prices
$p^i \in [0, \frac{1}{3}]$	L	L	$\{ \frac{1}{6}, \frac{1}{6} \}$
$p^i \in [\frac{1}{3}, \frac{2}{3}]$	H	M	$\{ \frac{2}{3}, \frac{1}{2} \}$
$p^i \in [\frac{2}{3}, 1]$	H	H	$\{ \frac{2}{3}, \frac{5}{6} \}$

Table 5: Action Space

We solve the optimization problem by backward induction since we are considering a farsighted approach. If the boss does not learn anything about the type of his namesake during the first period, he starts second period with the belief that both types are equally likely. Following the same reasoning of the previous section, we get the following reaction function:

$$R(p_1^2 |\, type1 \,\&\, type2) = \begin{cases} \text{report } p^2 \in [0, \frac{1}{3}] & \text{if } p_1^2 \in [\frac{1}{3}, \frac{23}{60}], \\ \text{report the true price} & \text{otherwise,} \end{cases} \tag{2}$$

where $R(p_1^2|\, type1 \,\&\, type2)$ stands for the reaction function for the second period given that both type-1 and type-2 are among possible types. And the expected loss in this case is

$$E_2(L|\, type1 \,\&\, type2) = \frac{151}{17280}, \tag{3}$$

where $E_2(L)$ denoted the expected loss in the second period. If the boss learns that his namesake is of type-1 during the first period, his reaction function for the second period is

$$R(p_1^2|\, type1) = \begin{cases} \text{report } p^2 \in [0, \frac{1}{3}] & \text{if } p_1^2 \in [\frac{1}{3}, \frac{25}{60}], \\ \text{report the true price} & \text{otherwise.} \end{cases} \tag{4}$$

The expected loss in this case is

$$E_2(L|\, type1) = \frac{7}{576} \tag{5}$$

If the boss learns that his namesake is of type-2 during the first period, his reaction function for the second period is to always report the true value, since given the number of categories and the distribution of the price this type uses optimal categorization. In this case, the expected loss is

$$E_2(L|\, type2) = \frac{1}{216} \tag{6}$$

Now, we move to the first period. If the boss, after observing the price on the first market, reports $p^1 \in [0, 1/3]$ then both types will use low category and 1/6 as representative price for the first market (see Table 5). Since both types will be using the same representative, they will behave in the same way. Therefore, it will be impossible for the boss to distinguish between the two, i.e., the boss will not learn anything about the type of his namesake and will continue with his initial belief. In this case, the overall expected loss will be the sum of the expected loss from the first period and the expected loss of the second period multiplied by the discount factor of the boss, $\delta \in [0, 1]$.

$$\int_{p_1^1}^{1/6} (p_2^1 - p_1^1)\, dp_2^1 + \delta\, E_2(L|\, type1 \,\&\, type2). \tag{7}$$

If the boss reports $p^1 \in [1/3, 2/3]$ then type-1 will use his high category and 2/3 as representative price for the first market, whereas type-2 will use his medium category and 1/2 as representative (see Table 5). If the price realization on the second market is below 1/2 then both types will act in the same way and will buy from second market. If it is greater than 2/3 both types will act again in the same way and will buy from the first market. However, if $p_2^1 \in [1/2, 2/3]$, type-1 will buy from the second whereas type-2 will buy from the first market. Thus, the boss will learn the exact type of his namesake only if $p_2^1 \in [1/2, 2/3]$ and this occurs with probability 1/6. Hence, with this strategy the probability that he boss figures out that his namesake is of type-1 is $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$, which is the same for type-2. In this case, the expected loss is

$$\frac{1}{2}\int_{p_1^1}^{2/3}(p_2^1-p_1^1)\,dp_2^1+\frac{1}{2}\int_{p_1^1}^{1/2}(p_2^1-p_1^1)\,dp_2^1+\delta\Big[\frac{5}{6}E_2(L|type1\&type2)+\frac{1}{12}E_2(L|type1)+\frac{1}{12}E_2(L|type2)\Big] \tag{8}$$

If the boss reports $p^1 \in [2/3, 1]$ then type-1 will use his high category and $2/3$ as representative price for the first market, whereas type-2 will also use his high category and $5/6$ as representative (see Table 5). In this case, both types will act in the same way unless the price realization on the second market belongs to $[2/3, 5/6]$. Thus, the boss will learn the exact type of his namesake only if $p_2^1 \in [2/3, 5/6]$ and this occurs with probability $1/6$. Hence, the expected loss in this case is

$$\frac{1}{2}\int_{p_1^1}^{2/3}(p_2^1-p_1^1)\,dp_2^1+\frac{1}{2}\int_{p_1^1}^{5/6}(p_2^1-p_1^1)\,dp_2^1+\delta\,[\frac{5}{6}E_2(L|type1\&type2)+\frac{1}{12}E_2(L|type1)+\frac{1}{12}E_2(L|type2)] \tag{9}$$

Inserting (3), (5) and (6) into (7), (8) and (9) we derive the reaction function of the boss as follows:

$$R(p_1^1) = \begin{cases} \text{report } p^1 \in [0, \frac{1}{3}] & \text{if } p_1^1 \in [0, a], \\ \text{report } p^1 \in [\frac{1}{3}, \frac{2}{3}] & \text{if } p_1^1 \in [a, \frac{2}{3}], \\ \text{report } p^1 \in [\frac{2}{3}, 1] & \text{if } p_1^1 \in [\frac{2}{3}, 1], \end{cases} \tag{10}$$

where $a = \frac{2760-\delta}{7200}$.

Taking into account the assumption that the boss prefers to tell the truth whenever it is among the optimal actions, the above reaction function becomes

$$R(p_1^1) = \begin{cases} \text{report } p^1 \in [0, \frac{1}{3}] & \text{if } p_1^1 \in [\frac{1}{3}, a], \\ \text{report the true price} & \text{otherwise.} \end{cases} \tag{11}$$

The reaction function (11) shows that the optimal strategy depends on the discount factor of the boss. If the boss concentrates only on the expected loss of the current period and does not take into account future expected losses, i.e., if $\delta = 0$, then myopic and farsighted optimization results coincide. Whenever the boss considers current losses together with future losses (i.e., whenever $\delta \neq 0$), there is a difference, albeit small, between the reaction functions resulted from myopic and farsighted approach.

Here we consider a game with only two periods. Before the last, there is only one period in which the boss can learn something about the type of his namesake. Furthermore, he has only one period, namely the second period, where he can use this information. This is the reason why the difference between reaction functions resulting from myopic and farsighted optimizations is so small. This difference is increasing in the number of periods of the game as well as in δ. A boss with high δ is more concerned about future loss compared to a boss with lower δ. Therefore he is more willing to invest in learning the type of his namesake in order to decrease his future loss.

4 Conclusion

We have constructed a model in order to study the interaction between fully and boundedly rational agents when they are parts of the same team and have perfectly aligned preferences. In an environment with abundance of information (type, quality, brand, age of the good, length of the warranty, payment arrangements and service fees), boundedly rational agents are having difficulties in making decision due to their cognitive limitations. In order to simplify the situation, they try to group events, objects or numbers into categories. In our model we consider a boundedly rational agent who partitions the price space into connected sets. The decision made by this agent might be non-optimal in some cases, since he is using categories instead of realized prices and regards prices belonging to the same category as equal.

Assuming different types for the boundedly rational agent and that types differ only in categories they use, we show that during his interaction, the fully rational agent may learn about the type of the boundedly rational agent, and using this additional information, he can improve the outcome. The probability that he learns the type of the boundedly rational agent increases in the length of this interaction, whereas it decreases in the number of available types.

Finally, we show that myopic and farsighted approaches yield different results in some cases, depending on the available types. This difference is caused by the tradeoff between experimenting for the future and starting to cope with the problem right away.

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